Optimal Mechanisms for Selling Information

Moshe Babaioff
(MSR-SVC)

Robert Kleinberg
(Cornell)

Renato Paes Leme
(Cornell)

The Secret Agent

The Secret Agent

Moscow

London

Valencia

The Secret Agent

The Informant

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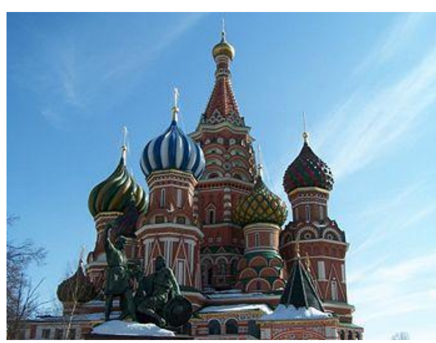

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How to sell information ?

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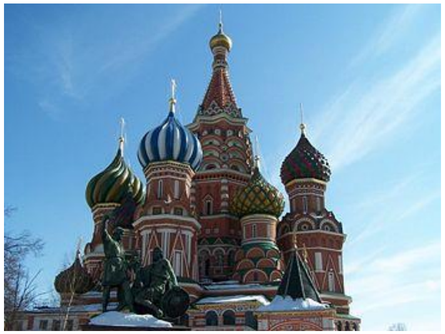

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This is not (only) a talk about espionage.

cookies,
user data…

The Information

How to sell information ?

The Advertiser

The Data Provider

Potential ads to show

The Information

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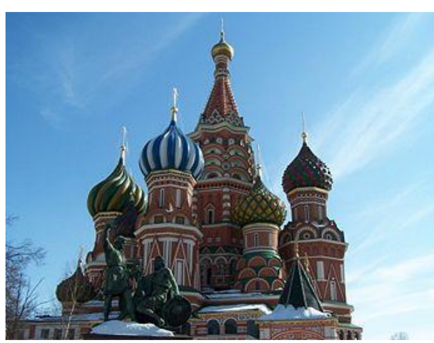

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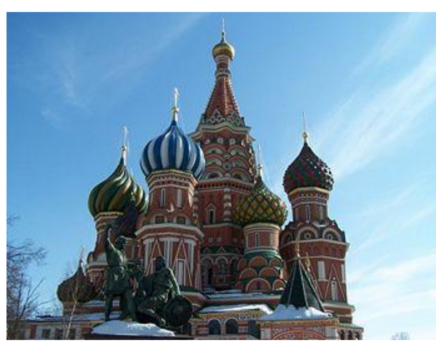

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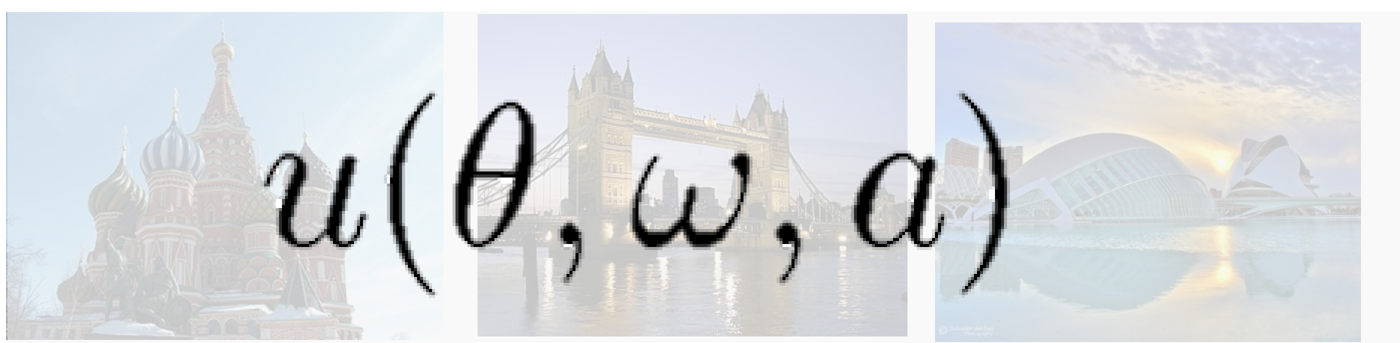

$$u(\theta, \omega, a)$$

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$$\boxed{\omega}$$

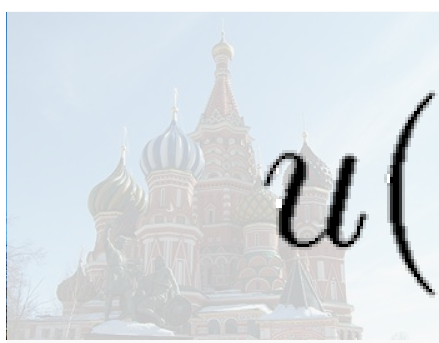
The Information

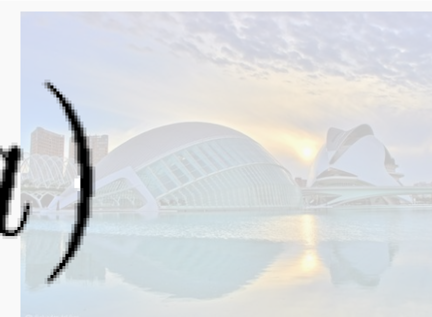
The Secret Agent

Common Bayesian Prior

$$(\omega, \theta) \sim \mu$$

The Informant





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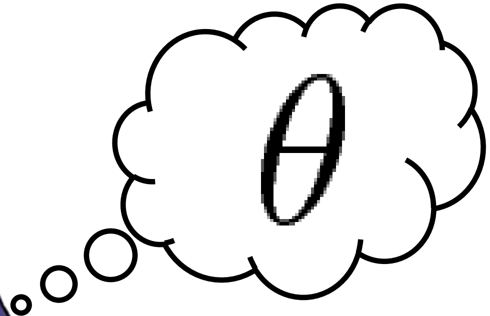

θ

The Buyer

TOP SECRET

ω

The Information

The Seller

Common Bayesian Prior

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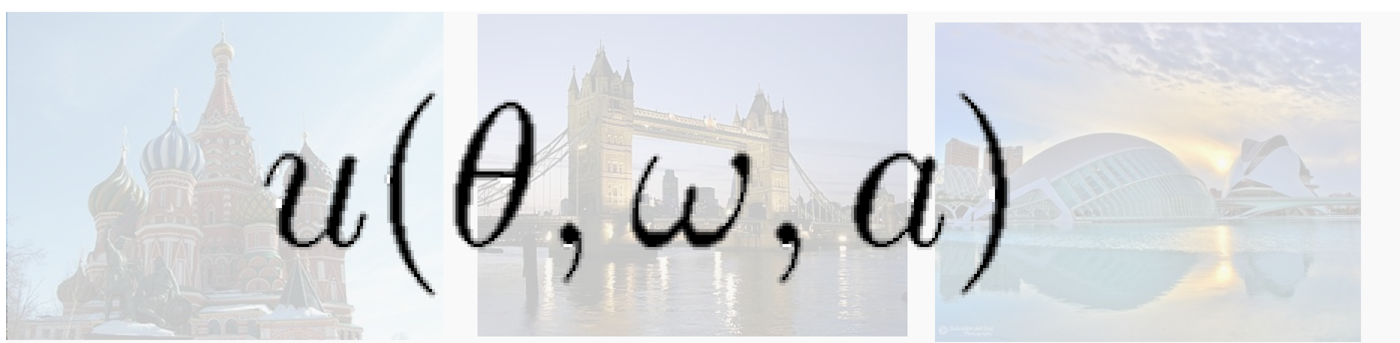

$$u(\theta, \omega, a)$$

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More formally …

- Seller knows ω. Buyer knows θ.
- Pair (ω, θ) comes from a joint distribution $\mu(\theta, \omega)$ that is common knowledge
- Buyer needs to pick an action $a \in A$ getting reward $u(\theta, \omega, a)$

$\boxed{\textbf{Context:} \quad (\mu, u)}$

Buyer (Secret Agent) Utility

- If he doesn't know ω (i.e. only knows θ)

$$U = \max_{a \in A} \mathbb{E}[u(\omega, \theta, a) | \theta]$$

- If he also knows ω

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- Expected surplus for full information

$$\xi_\theta = \mathbb{E}[\max_{a \in A} u(\omega, \theta, a)|\theta] - \max_{a \in A} \mathbb{E}[u(\omega, \theta, a)|\theta]$$

How much of this surplus can the seller (informant) extract given that he doesn't know θ ?

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- $\theta \in \{H, L\}$ with ½ probability (danger level)

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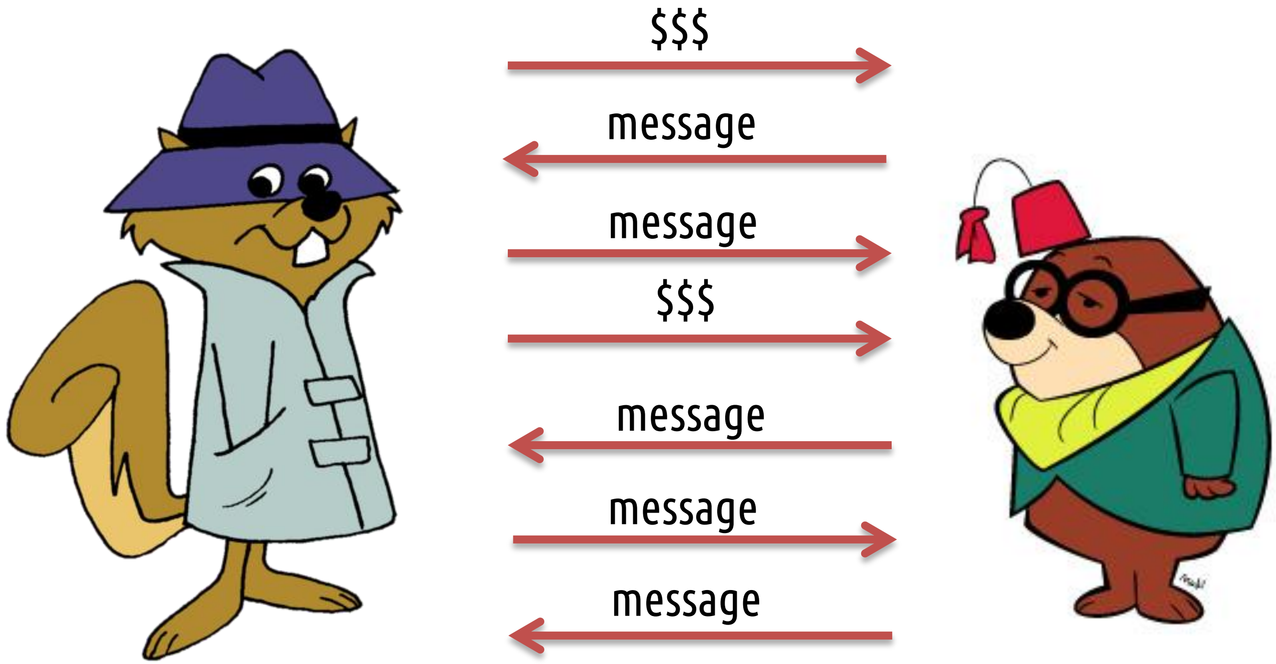
Information is a lot more flexible than traditional goods.

What is a feasible mechanism ?



Informant proposes a mechanism based on (μ, u)
and commits to faithfully follow it.
The agent is strategic.
Informant wants to maximize revenue.

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Independent ω and θ

Theorem: If ω and θ are independent, there exists an optimal mechanism that offers to the buyer a list

$$(Y_{\theta_1}, t_{\theta_1}), \ldots, (Y_{\theta_n}, t_{\theta_n})$$

where Y_θ is a random variable correlated with ω and t_θ is its price.

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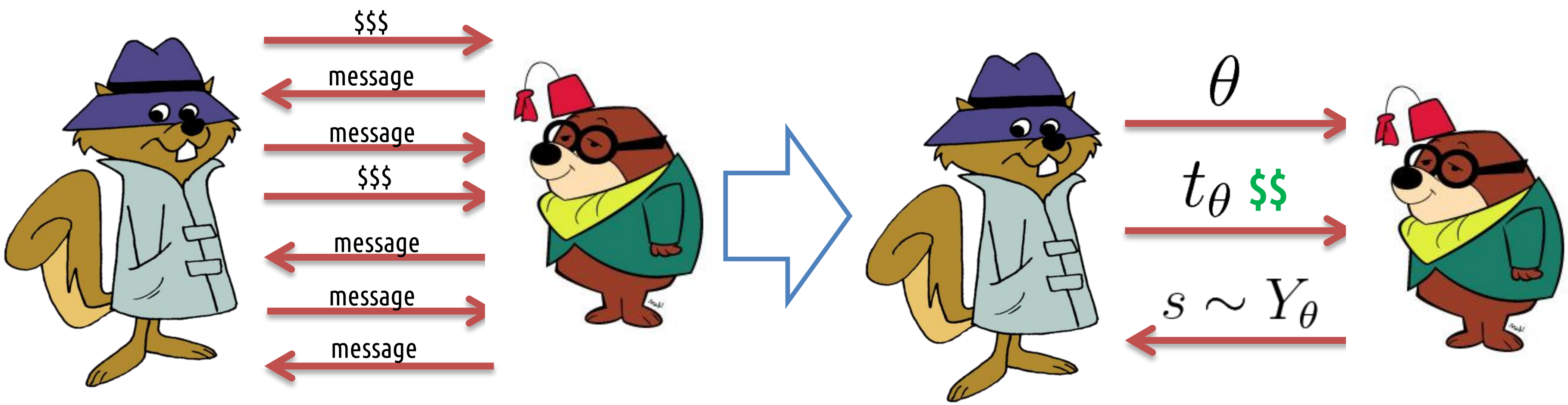
Examples: Y_i is a “noisy” version of ω :

- a subset of the bits
- the XOR of two bits
- ω with prob ½ and random with prob ½

Independent ω and θ

What does this theorem mean?



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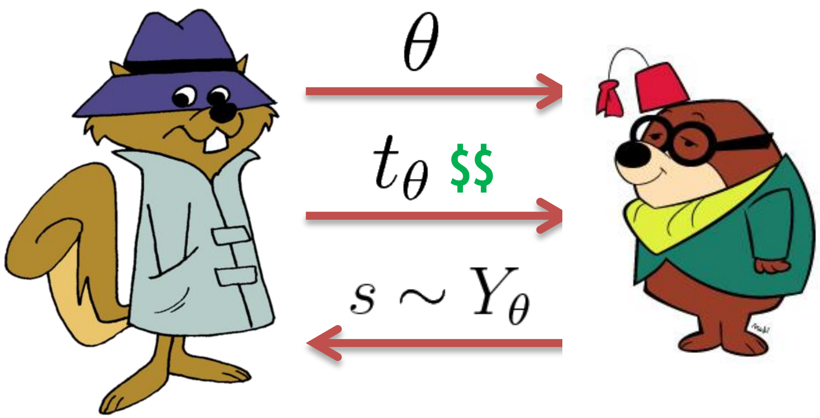
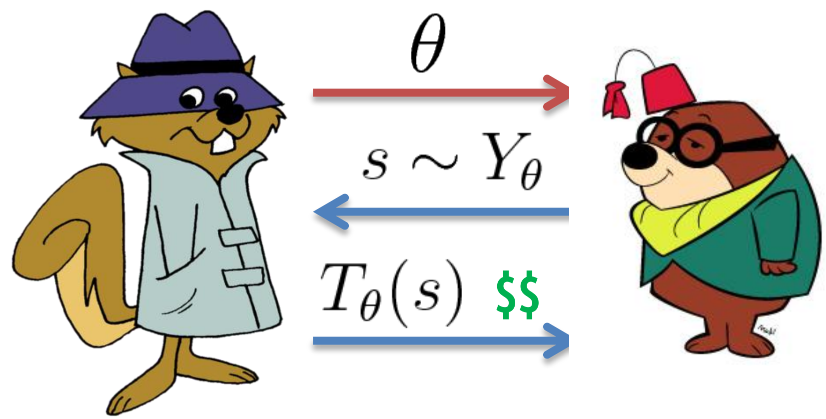
We can find the optimal mechanism in polynomial time using convex programming.

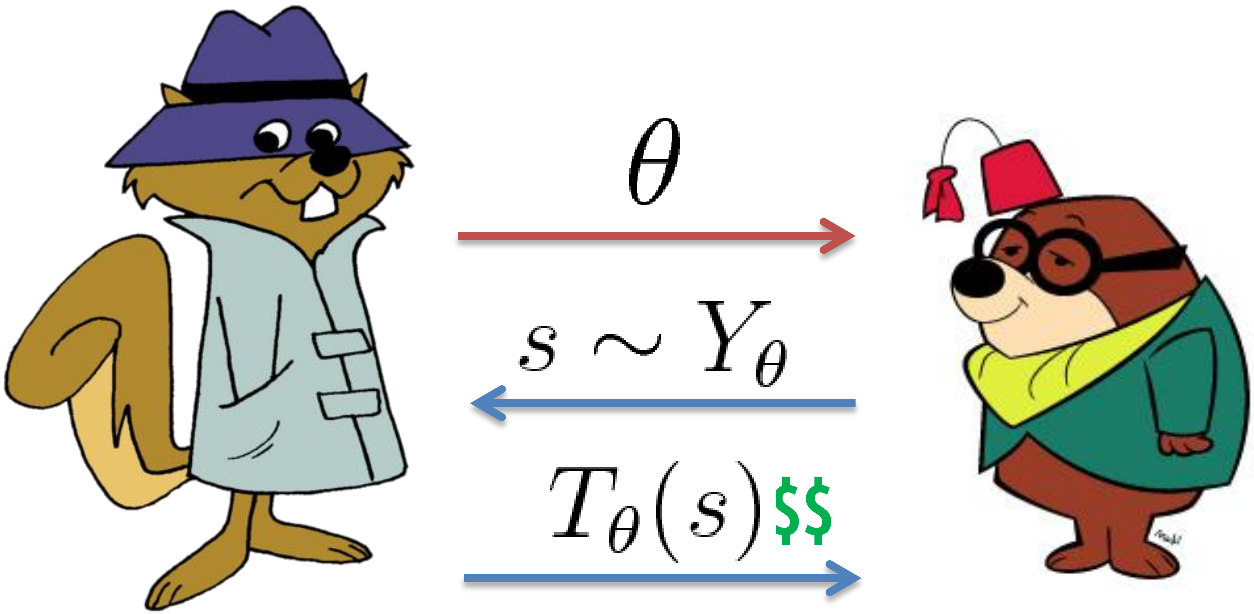


Independent case



Correlated case

What bad can happen ?



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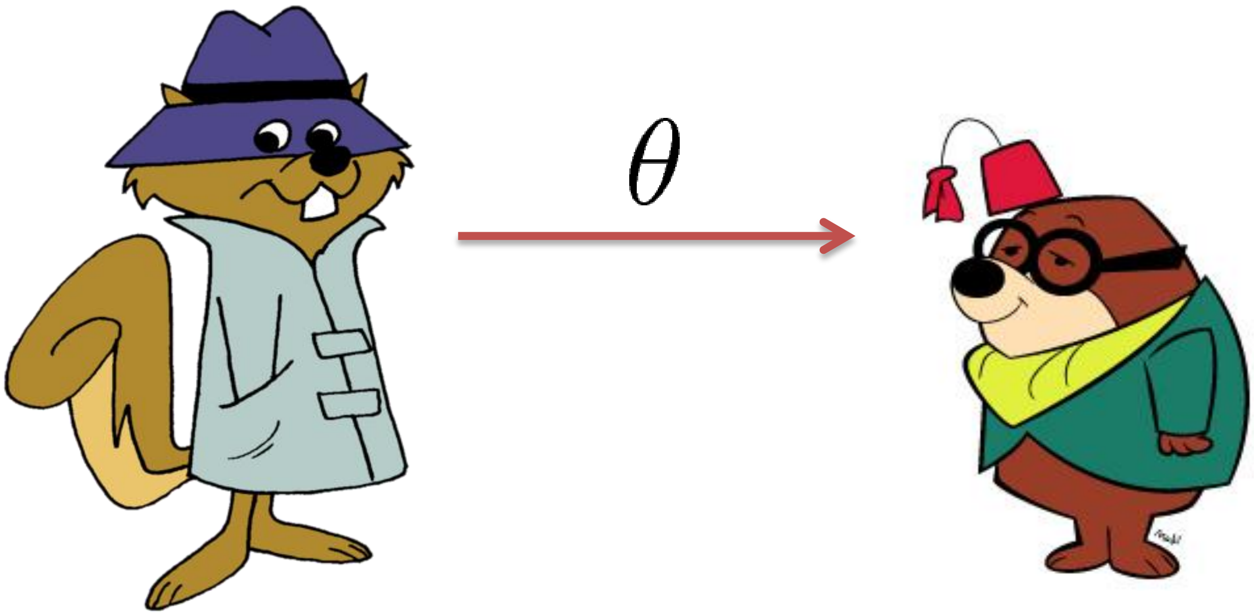

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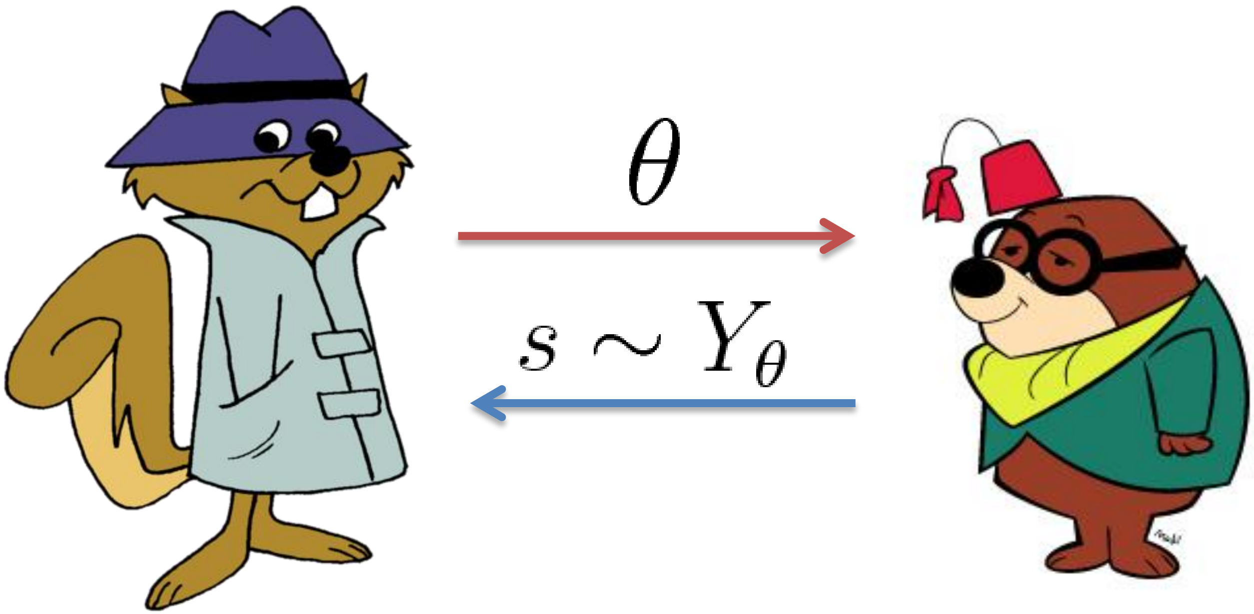

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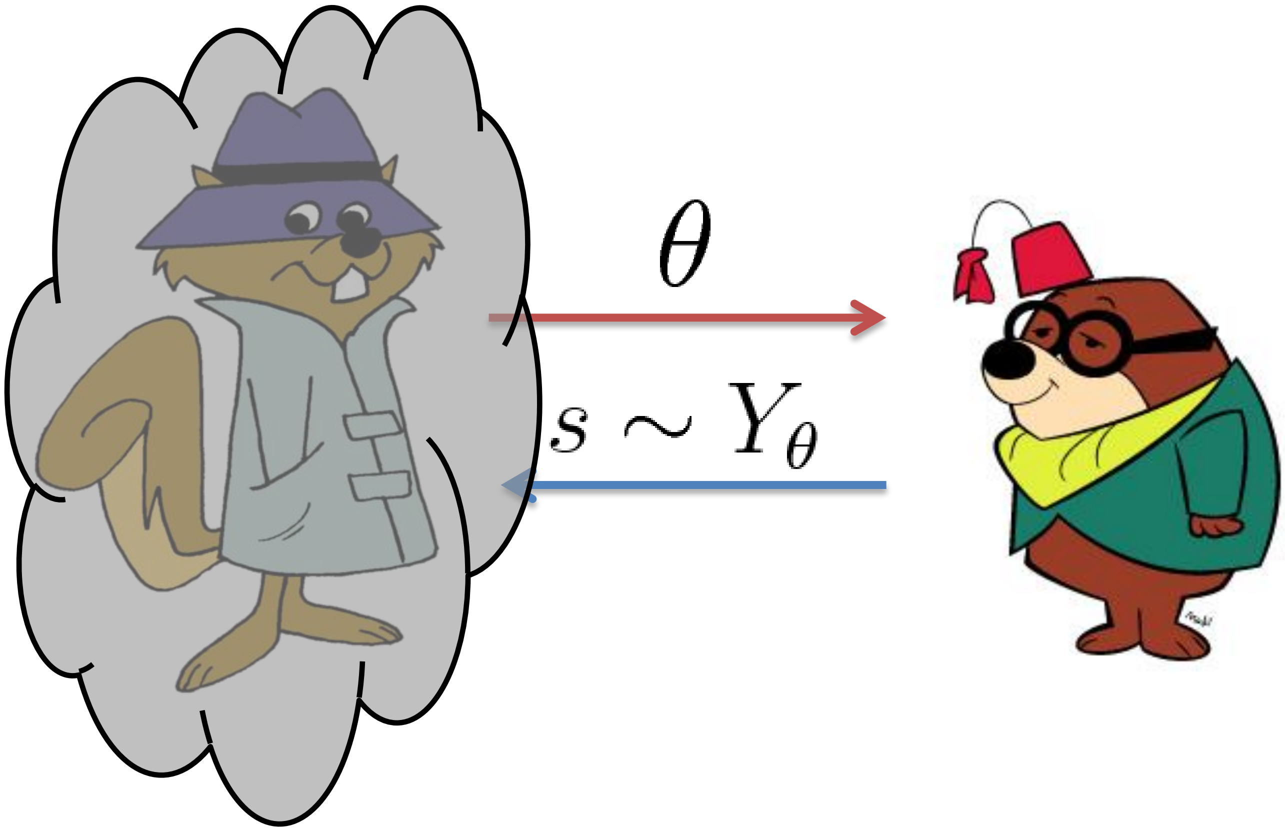

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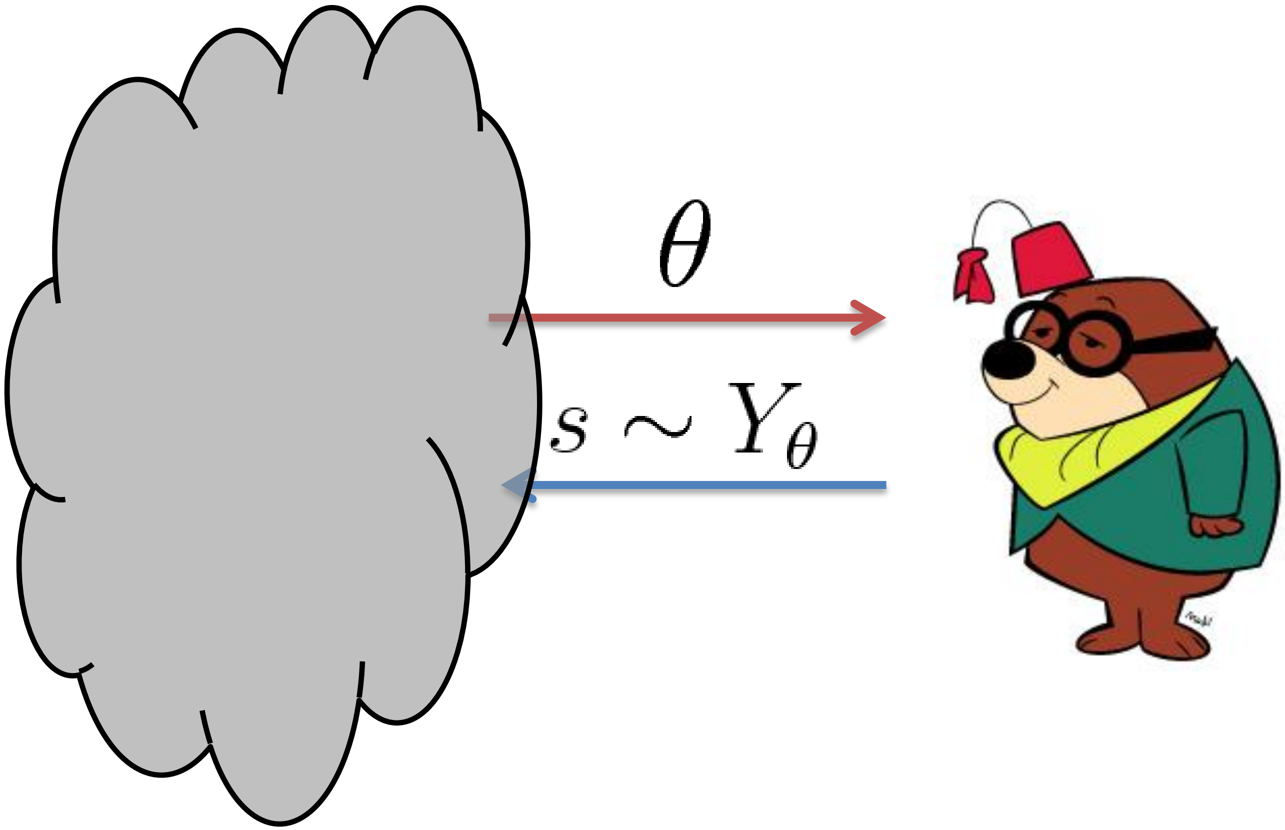

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$$\xrightarrow{\quad\theta\quad}$$

$$\xleftarrow{\quad s \sim Y_\theta\quad}$$

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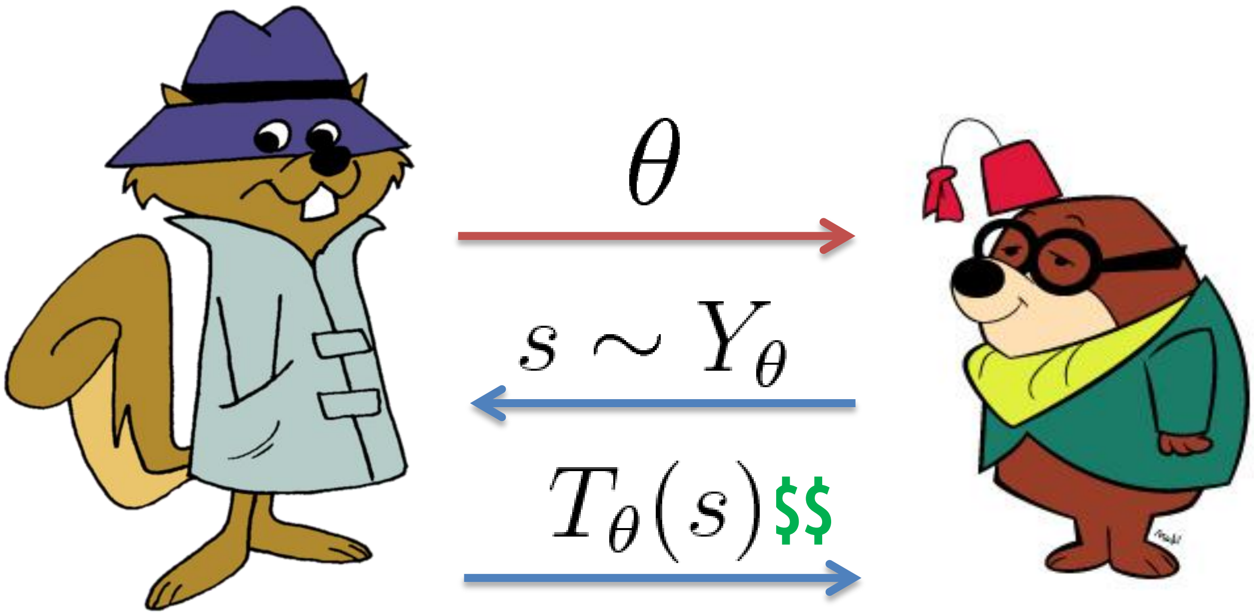
This mechanism doesn’t work if the buyer is allowed to defect at any point.

One possible fix : large deposit upfront



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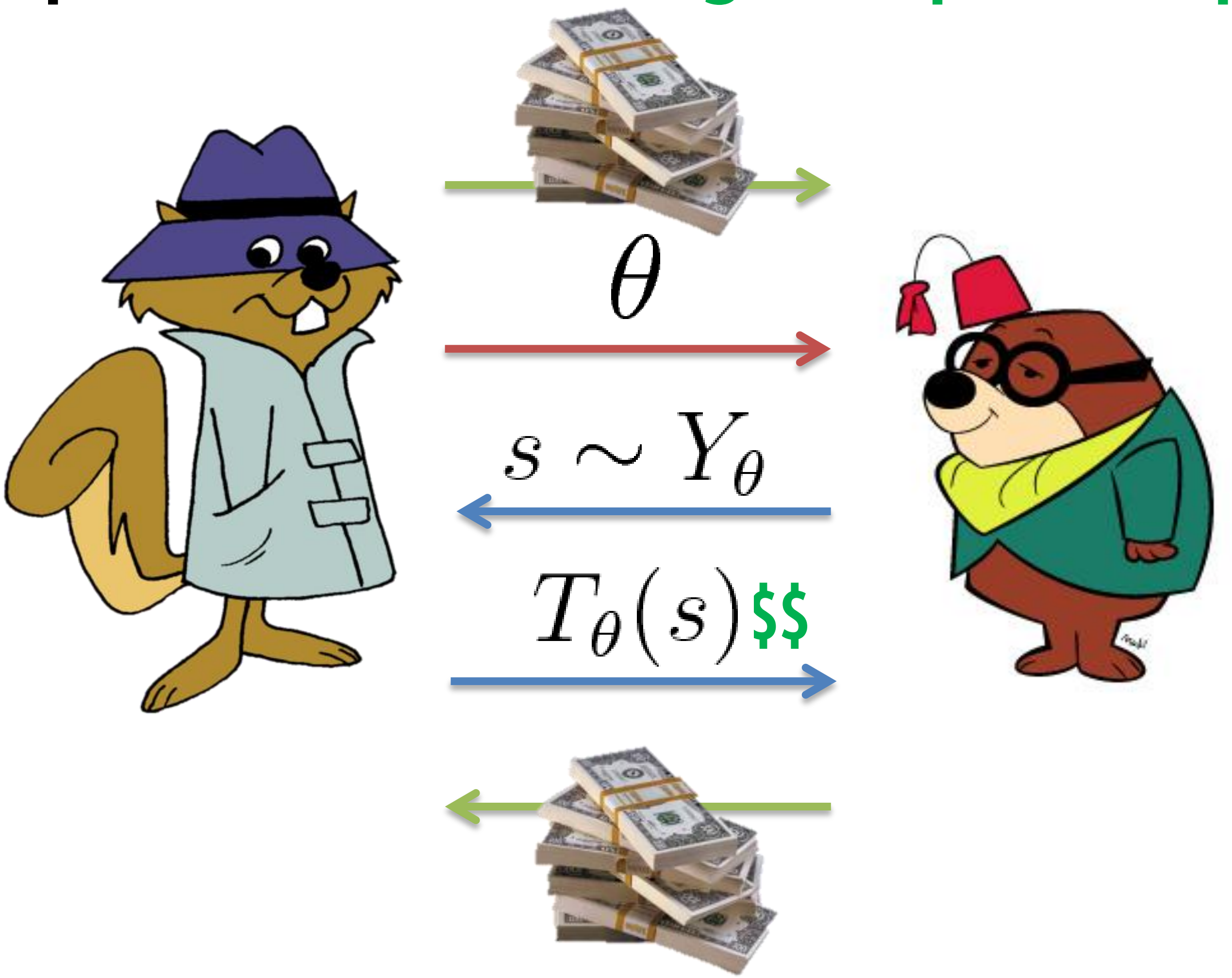

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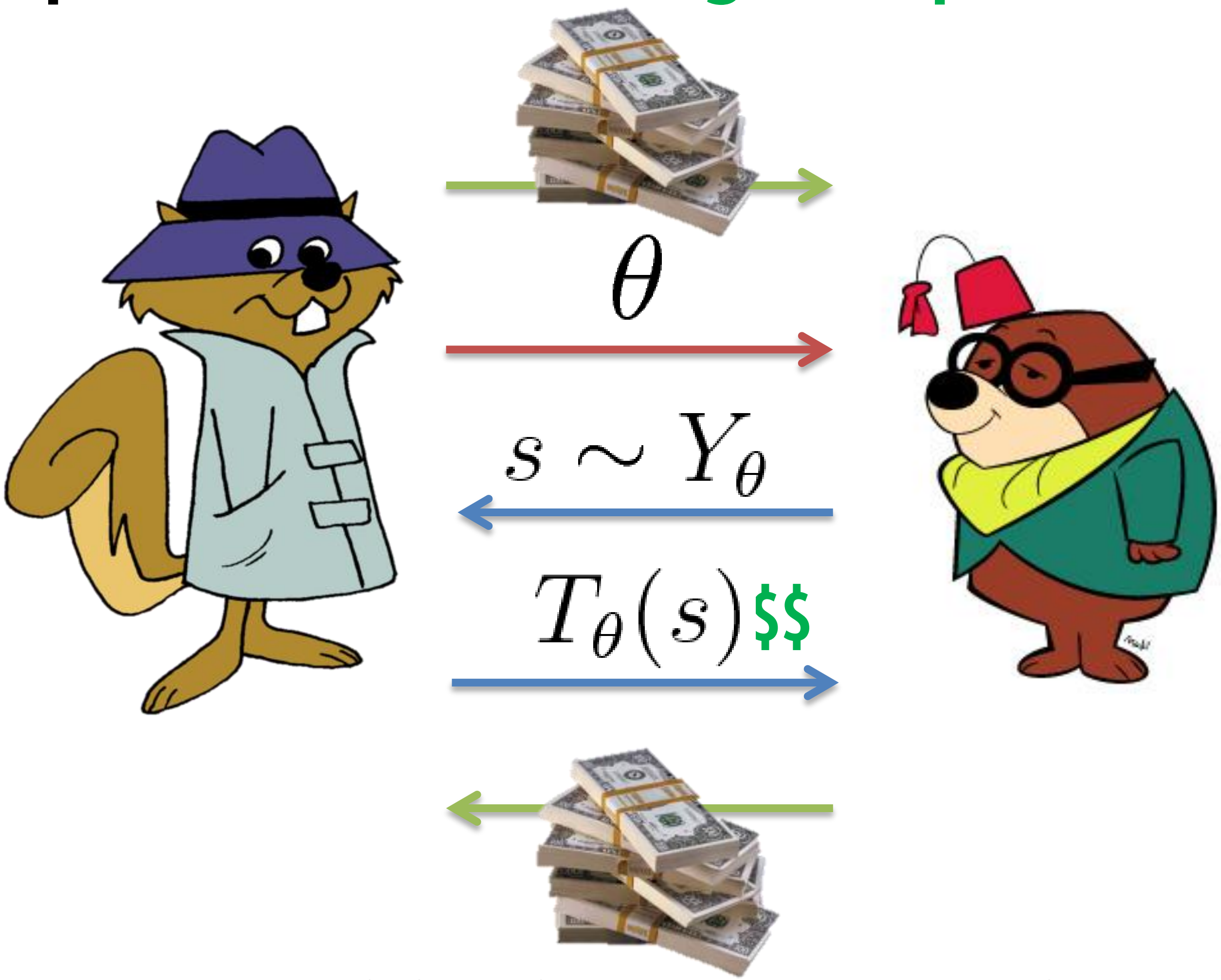

Increases participation cost, creates incentives for the informant to defect, …

Question:

What is the revenue optimal mechanism where
(1) buyer is allowed to defect
(2) no positive transfers are allowed

The answer is **puzzling**.

Mechanisms for uncommitted buyers with no positive transfers

Theorem: Interactive mechanism are necessary in order to get optimal revenue.

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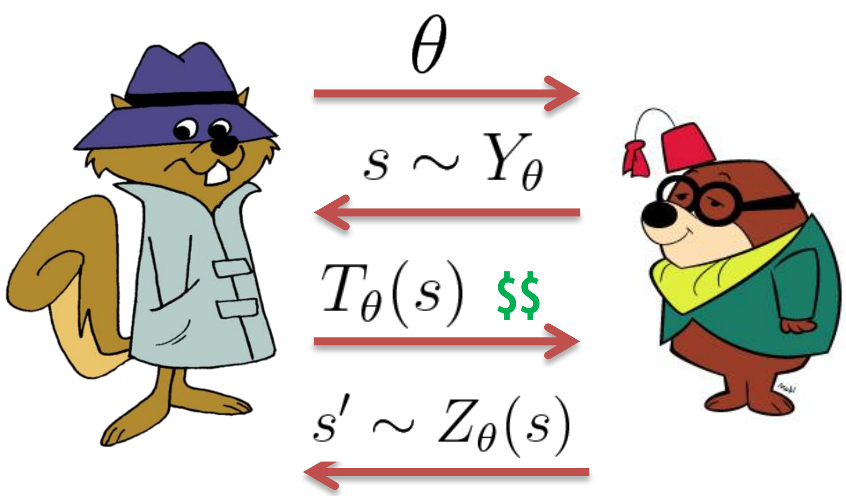

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How to optimize over interactive mechanisms ?

How to do mechanisms design beyond the one-round revelation principle ?

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Open Problems

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Multiple buyers and sellers : a market for information

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Continuous type spaces